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Covariance Compensation Extended Kalman Filter를 이용한 GPS에서의 동적 추정

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Dynamic Estimation in GPS through Covariance Compensation Extended Kalman Filter

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ABSTRACT

This paper presents a covariance compensation extended Kalman filter(CCEKF) based approach to navigation using the Global Positioning System(GPS). The covariance compensation is used to decrease the effect of unexpected measurement and process uncertainties. This paper relies on a detailed modeling of GPS using the data generated with constant velocity through Yuma Almanac.

Key words : global positioning systems, extended Kalman filter, navigation, filtering

I. INTRODUCTION

GPS is a satellite-based navigation system that allows a user with the proper equipment access to useful and accurate positioning information anywhere on the globe[1,2].

The GPS satellites are uniformly distributed in a total of six orbits such that there are four satellites per orbit. This number of satellites and spatial distribution of orbits insures that at least eight satellites can be simultaneously seen at any time from almost anywhere on Earth. The GPS satellites circle the Earth at an altitude of about 20,000(13,000 miles) and complete two full orbits every day. The GPS satellites are not in a geostationary orbit, but rise and set two times per day. Each satellite broadcasts radio waves towards Earth that contain information regarding its position and time. We can receive this information by using special receivers, called GPS receivers, which can detect and decode this information. By combining signals transmitted by several satellites and received simultaneously, a GPS receiver can calculate its position on the Earth.

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For nearly three decades, the position estimation problem in GPS has been a fruitful application area for state estimation. Many problems have been solved, yet new and diversified applications still challenge system engineers. Various dynamic models[3,4] for GPS positioning have been proposed over the years, differing in their complexity. These models include Position(P) model, Position-Velocity(PV) model and Position-Velocity-Acceleration(PVA) model. Optimization filters like extended Kalman Filter(EKF) have been proposed to solve these estimation problems.

Since state space model is used, the EKF is derived from process equation and non-linear measurement equation having process noise and measurement noise respectively. The GPS positioning problem is closely related to target tracking problem. An earlier reference on tracking-trajectory estimation[5] deals with tracking algorithms for ballistic reentry vehicles. The estimation problem is a problem of nonlinear estimation. A rigorous treatment of the nonlinear estimation requires the use of stochastic integrals and differential equations. An excellent paper[6] presents a survey of problems and solutions in the area of target tracking. It discusses design tradeoffs, performance evaluation and techniques for optimizing state estimation problem. It also discusses four approaches for tracking targets with sudden maneuvers. Basing our research on this, we have worked extensively on both modeling and measurement uncertainties and have proposed a CCEKF based approach to estimation which also caters to unexpected process and measurement uncertainties that may arise in the system due to faulty measurements and also in those cases where for a short period of time the filter does not follow the dynamic model strictly i.e. sudden maneuvers. Surely, the EKF algorithm can be improved by iterated EKF and nonlinear second order filter as suggested in [1] and [2] but they aid in optimality rather than circumventing unexpected uncertainties.

The remainder of this paper is organized as follows. In the next section the GPS positioning problem is formulated mathematically. This includes dynamic state process model and measurement model. Previous approaches for enhancing EKF are discussed in section III. The proposed idea is discussed in section IV and simulation study, which was performed in order to assess the merits of the proposed approach for enhancing EKF is presented in section V. Concluding remarks are offered in the final section. Appendix is provided at the end to discuss different types of dynamic models.

II. PSEUDOMEASUREMENT EQUATION

A GPS receiver fundamentally measures a quantity called pseudorange ρ, which is a raw, on-way range measurement corrupted by a user clock bias. Using either models or measurements, ρ can be corrected for atmospheric effects to produce corrected pseudorange. With an approximate user location, the receiver can then process these corrected pseudo-ranges(to four or more satellites) to determine location in a convenient coordinate system.

For GPS, the underlying coordinate system is currently the 1984 World Geodetic System (WGS-84), which is an accepted worldwide geodetic coordinate system. It is usual and convenient for the receiver to perform initial calculations in an Earth Centered Earth Fixed (ECEF) Cartesian coordinate system.

The pseudo-measurement equation for GPS for ith satellite can be defined as

$$\rho^i = \sqrt{(X^i - x)^2 + (Y^i - y)^2 + (Z^i - z)^2} + c\Delta t_r + c\Delta t_{sv} + c\Delta t_{ion} + c\Delta t_{tropo} + \eta \qquad (1)$$

where $c\Delta t_r$ is range correction due to receiver clock offset, $c\Delta t_{sv}$ is range correction due to satellite clock offset, $c\Delta t_{ion}$ is range correction due to ionospheric error, $c\Delta t_{tropo}$ is range correction due to tropospheric error, is ECEF coordinate for the ith satellite and η are other errors. For detailed information, see[1-2].

III. PROBLEM DEFINITION

The problem is a state estimation problem, i.e., assuming the state of the target evolves in time according to the equation.

$$\zeta_{k+1} = \phi_k \zeta_k + w_k \qquad (2)$$

and the corresponding nonlinear measurement vector is given by

$$\zeta^*{}_k = H_k(\zeta_k) + v_k \qquad (3)$$

where w_k and v_k are input and measurement noise processes respectively, one is interested in estimating the target states ζ_k based upon all measurements $\rho_k^* = 1, ..., k$. For details on the dynamic models, see the appendix.

Eq. (2) is a mathematical model representative of the target dynamics. The state vector ζ_k usually contains target position, velocity, and sometimes acceleration as state variables along with receiver clock offset error range. Eq. (3) is the measurement equation relating state variables to measurement variables. Here ρ_k^* contains at least four satellite ranges ρ_k measured from the receiver in addition to the range errors resolved from Eq. (1).

The system(input) and measurement noise processes w_k and v_k are assumed to be zero mean white noise processes. The covariance of w_k, Q is selected to compensate for modeling errors. The covariance of the measurement noise v_k, R should also be selected to present all the excursions such as measurement biases, false measurements, etc.

Since $H_k(\zeta_k)$ is nonlinear, we can linearize it to the previous predicted state and solve it recursively with EKF. In such case we define:

$$\rho_k^* = H_k(\zeta_{k|k-1}) + J_k(\zeta_{k|k-1})(\zeta_k - \zeta_{k|k-1}) + \nu_k \qquad (4)$$

We can define pseudo-measurement as

$$y_k \equiv \rho_k^* - H_k(\zeta_{k|k-1}) + J_k(\zeta_{k|k-1})\zeta_{k|k-1} \qquad (5)$$

Hence,

$$y_k = J_k(\bar{x}_{k|k-1})\bar{x}_k + \nu_k \qquad (6)$$

The EKF algorithm then works by minimizing the cost functional defined as:

$$C(\zeta_{k|k}) = \frac{1}{2}\left\{ \begin{matrix} \|y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}\|_{(R_k)^{-1}} \\ + \|\zeta_{k|k} - \zeta_{k|k-1}\|_{(P_{k|k-1})^{-1}} \end{matrix} \right\} \qquad (7)$$

The EKF equations are given as follows:

Measurement Update Step(Filtering)

$$K_k = P_{k|k-1}J_k^T[J_k(\zeta_{k|k-1})P_{k|k-1}J_k(\zeta_{k|k-1})^T + \alpha R_k]^{-1} \quad (8)$$

$$\zeta_{k|k} = \zeta_{k|k-1} + K_k(y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}) \quad (9)$$

$$P_{k|k} = (I_N - K_k J_k(\zeta_{k|k-1}))P_{k|k-1} \quad (10)$$

Time Update Step(Prediction)

$$P_{k+1|k} = \phi_k P_{k|k}\phi_k^T + Q_k \quad (11)$$

$$\zeta_{k+1|k} = \phi_k \zeta_{k|k} \quad (12)$$

IV. RELATED WORK

The previous work done on targets with sudden maneuvers modeled as systems with abrupt changes for target tracking systems. In [7], filter compensation using process noise covariance is proposed. In this method, the estimator's only concern is to maintain the target in track(adequate position estimation accuracy), this method can work quite well. Basically it examines the "regularity" of the filter residual vector.

$$\gamma_k = y_k - H(\hat{\zeta}_{k|k-1}) \quad (13)$$

against its covariance matrix

$$P_{\gamma_k} = J_k(\zeta_{k|k-1})P_{k|k-1}J_k^T(\zeta_{k|k-1}) + R_{\gamma_k} \quad (14)$$

using(the chi-square variable)

$$I_k = \gamma_k^T P_{\gamma k}^{-1}\gamma_k \quad (15)$$

When Eq. (15) becomes too large one suspect that the target is maneuvering and the covariance Q is increased so that I_k is reduced to a reasonable value. This method therefore has the combined feature of maneuver detection and filter compensation. This method is based on the adaptive filter of Jazwinski and shares great similarities with the method of Mehra[8] which is a method for testing filter optimality based upon the innovation process is mentioned. If the test indicates that the filter does not attain optimal performance, one then proceeds to adjust Q and R so that the covariance of the innovation process will be consistent with that of filter prediction. This method first computes a sampled correlation function assuming that γ_k is ergodic over a certain time interval, one therefore has

$$C_j \cong E[\gamma_i\gamma_{i-1}^T] = JP_\infty J^T + R \quad ;j=0$$
$$= J[\phi(I-KJ)]^{j-1}\phi[P_\infty J^T - KC_0] \quad ;j>0 \quad (16)$$

and

$$\hat{C}_j = \frac{1}{N}\sum_{i=1}^{N}\gamma_i\gamma_{i-1}^T \quad (17)$$

where P_∞ denotes the steady-state error covariance matrix and ϕ is the state transition matrix of the system dynamics. Using the above equations and the steady-state Riccati equation, Mehra gives a procedure for solving for Q and R. There are situations in which there is no sufficient number of independent equations for solving them; Mehra then gives a recursive procedure for solving for the filter gain K directly.

A more recent compensation filter[9] proposed is nonlinearity-compensation extended Kalman Filter(NLCEKF). For NLCEKF, the cost functional is defined as

$$C(\zeta_{k|k}) = \frac{1}{2}\left\{ \begin{array}{l} \|y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}\|_{(R_k)^{-1}} \\ + \|\zeta_{k|k} - \zeta_{k|k-1}\|_{(P_{k|k-1})^{-1}} \end{array} \right\} \tag{18}$$

The NLCEKF recursive equations are the same as described in[8-12] except for the two changes

$$\zeta_{k|k} = \zeta_{k|k-1} + \chi K_k(y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}) \tag{19}$$

and

$$P_{k|k} = (I_N - \eta^2 K_k J_k(\zeta_{k|k-1}))P_{k|k-1} \tag{20}$$

Here

$$\eta = \begin{cases} \chi & : 0 \le \chi \le 1 \\ 2-\chi & : 1 \le \chi \le 2 \end{cases} \tag{21}$$

Let us consider the meaning of coefficient χ here. When $\chi = 1$, Eq. (19) is the same as EKF. If the cost function of Eq. (18) is the minimum when $\chi = 1$, this means that the nonlinearity of the system is small, and that error caused by linear approximation is small. If the position where the cost function of Eq. (18) becomes the minimum is distant from $\chi = 1$, then the error caused by linear approximation is large. When $\chi = 0$, the state is not updated even if new measurements are input. This means that it is sufficient to search the position where the cost function becomes the minimum in the range of $0 \le \chi \le 2$, with $\chi = 1$ at the center. The update value of the error covariance matrix of Eq. (20) is $\eta = 1$ when $\chi = 1$, and η is decreased as χ becomes distant from 1.

V. PROPOSED COMPENSATION FILTER

We modify the EKF cost functional by including the compensation parameters to the covariances as

$$R_k^* = \alpha R_k \tag{22}$$

and

$$P_{k|k-1}^* = \beta P_{k|k-1} \tag{23}$$

The cost functional for the modified case will become

$$C(\zeta_{k|k}) = \frac{1}{2}\left\{ \begin{array}{l} \|y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}\|_{(R_k^*)^{-1}} \\ + \|\zeta_{k|k} - \zeta_{k|k-1}\|_{(P_{k|k-1}^*)^{-1}} \end{array} \right\} \tag{24}$$

Here

$$\alpha = \left(\frac{\|y_k - J_k(\zeta_{k|k-1})\zeta_{k|k}\|}{\kappa} \right)^2 \tag{25}$$

and

$$\beta = \left(\frac{\|\zeta_{k|k} - \zeta_{k|k-1}\|}{\psi} \right)^2 \tag{26}$$

where κ is the reference residual between the true measurement and predicted measurement and ψ is the reference residual between the predicted states in consecutive iterations. These reference residuals can be obtained empirically through hit and trial method or alternatively, by obtaining the mean residual value for last N consistent iterations. The reason behind choosing α and β compensation factors is to make the filter resilient towards unexpected measurement and process uncertainties. These abrupt uncertainties can occur either because of sudden maneuvers or faulty measurements acquired for a brief time period. The novelty in our proposed algorithm is the adaptability of process error covariance matrix P and measurement noise covariance matrix R, which in the previous case was only restricted to process noise covariance matrix Q. We emphasize that the method presented above is for reducing performance sensitivities to noise. Furthermore, the previous strategies might only be useful for nonreal-time applications because of the computational requirements: minimizing J_k in Jazwinski's approach; minimizing ψ in the case of NLCEKF; Mehra's computationally intensive solution for optimizing Q and R.

VI. SIMULATIONS

In order to test CCEKF, synthetic data was generated by using Yuma almanac maintained at U.S Coast Guard Navigation Center[10]. From this data, ECEF coordinates of satellites were generated for 400 time steps with a sampling period of 10 seconds. Initial position was taken as 30o,128o latitude and longitude respectively for an object present on the surface of earth. Visible satellites were marked out. Circular path was generated for 400 time steps with the speed of object being 36 km/hr. PV model is used for simulation. White guassian noises were added as process noise and measurement noise. Two simulations were done: first to test the normal running of the filter and then to test the presence of unexpected measurement uncertainty. The reference residuals for the later cases were selected empirically. Fig. 1 and 2 represent the results obtained for formal case. Fig. 3 represents the comparison between EKF, NLCEKF and CCEKF in the presence of 10% unexpected measurement noise applied between 100th and 150th time step. In the presence of unexpected measurement and process noise, α and β is used to adjust the covariance by minimizing the effect of the noise. Hence a shift in root mean square error(RMSE) can be seen in Fig. 3. Here it can also be seen that CCEKF outperform NLCEKF algorithm in the presence of abrupt measurement and process uncertainties.

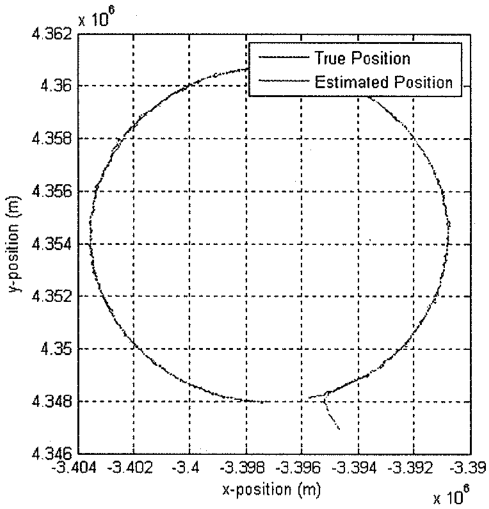

Fig. 1 Comparison between true position and position estimated through CCEKF

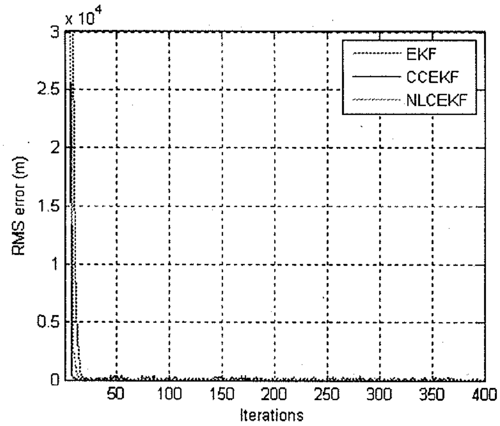

Fig. 2 RMS error between the true position and position estimated through CCEKF

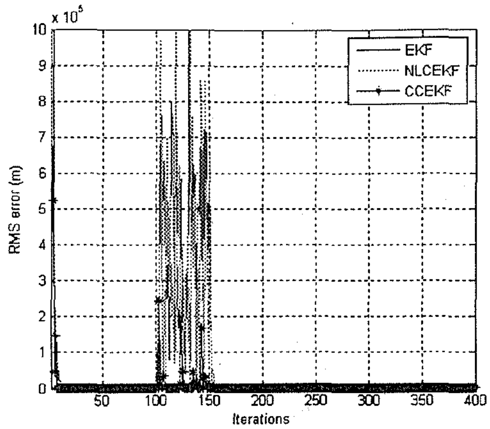

Fig. 3 Comparison between EKF, NLCEKF and CCEKF in the presence of unexpected measurement uncertainty

VII. CONCLUSIONS

For dynamic estimation in GPS, we have presented many strategies for improving the performance of extended Kalman filter and have proposed covariance compensation extended Kalman filter which is quite effective in those cases where the data is perturbed by unexpected measurement and process uncertainty. In those cases, conventional extended Kalman filter does not perform well. Advantage of covariance compensation extended Kalman filter is the presence of compensation term α and β that reduces the effect of unexpected measurement and process uncertainties. In comparison to the previously implemented compensation filters, we propose a light weight and cost-effective solution at the expense of a tolerable computational burden.

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APPENDIX: DYNAMIC MODELS FOR EKF

We define

$$H_k(\zeta_k) = \sqrt{(X^i - x_k)^2 + (Y^i - y_k)^2 + (Z^i - z_k)^2} + c\Delta t_r \tag{1}$$

Now consider three models for the analysis of GPS position accuracy. These three models correspond to stationary, low dynamic and high dynamic applications. In the case in which the receiver is known to be stationary, an position model(P) include only receiver position and clock bias states. In such case the state transition matrix ϕ_k is an identity matrix. The state vector is $\zeta_k = [x_k \ y_k \ z_k \ (c\triangle t_r)_k]^T$ and jacobian is defined as:

$$J_k(\zeta) = \begin{bmatrix} \frac{\partial \rho_k^1}{\partial x} & \frac{\partial \rho_k^1}{\partial y} & \frac{\partial \rho_k^1}{\partial z} & 1 \\ \frac{\partial \rho_k^2}{\partial x} & \frac{\partial \rho_k^2}{\partial y} & \frac{\partial \rho_k^2}{\partial z} & 1 \\ \frac{\partial \rho_k^3}{\partial x} & \frac{\partial \rho_k^3}{\partial y} & \frac{\partial \rho_k^3}{\partial z} & 1 \\ \frac{\partial \rho_k^4}{\partial x} & \frac{\partial \rho_k^4}{\partial y} & \frac{\partial \rho_k^4}{\partial z} & 1 \end{bmatrix} \tag{2}$$

When the receiver is in uniform motion(i.e., near constant velocity), performance is improved by the inclusion of velocity states in the model called Position-Velocity(PV) model. The state vector is $\zeta_k = [x_k \ y_k \ z_k \ (c\triangle t_r)_k]^T$. The state transition matrix and jacobian are defined as:

$$\phi_k = \begin{bmatrix} 1 & \triangle t & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \triangle t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \triangle t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

and

$$J_k(\zeta) = \begin{bmatrix} \frac{\partial \rho_k^1}{\partial x} & 0 & \frac{\partial \rho_k^1}{\partial y} & 0 & \frac{\partial \rho_k^1}{\partial z} & 0 & 1 \\ \frac{\partial \rho_k^2}{\partial x} & 0 & \frac{\partial \rho_k^2}{\partial y} & 0 & \frac{\partial \rho_k^2}{\partial z} & 0 & 1 \\ \frac{\partial \rho_k^3}{\partial x} & 0 & \frac{\partial \rho_k^3}{\partial y} & 0 & \frac{\partial \rho_k^3}{\partial z} & 0 & 1 \\ \frac{\partial \rho_k^4}{\partial x} & 0 & \frac{\partial \rho_k^4}{\partial y} & 0 & \frac{\partial \rho_k^4}{\partial z} & 0 & 1 \end{bmatrix} \tag{4}$$

where $\triangle t$ is sampling time.

When the velocity can not reasonably be modeled as constant, then the acceleration state can be added in each of the three orthogonal directions called position velocity (PVA) model. In such case the state transition matrix is defined as

$$\phi_k = \begin{bmatrix} 1 & \triangle t & \frac{1}{2}(\triangle t)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \triangle t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & & & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \triangle t & \frac{1}{2}(\triangle t)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \triangle t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \triangle t & \frac{1}{2}(\triangle t)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \triangle t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The state vector is

$$\zeta_k = [x_k \ \dot{x}_k \ \ddot{x}_k \ y_k \ \dot{y}_k \ \ddot{y}_k \ z_k \ \dot{z}_k \ \ddot{z}_k \ (c\triangle t_r)_k]^T \tag{6}$$

and jacobian is given as:

$J_k(\zeta) =$

$$\begin{bmatrix} \frac{\partial \rho_k^1}{\partial x} & 0 & 0 & \frac{\partial \rho_k^1}{\partial y} & 0 & 0 & \frac{\partial \rho_k^1}{\partial z} & 0 & 0 & 1 \\ \frac{\partial \rho_k^2}{\partial x} & 0 & 0 & \frac{\partial \rho_k^2}{\partial y} & 0 & 0 & \frac{\partial \rho_k^2}{\partial z} & 0 & 0 & 1 \\ \frac{\partial \rho_k^3}{\partial x} & 0 & 0 & \frac{\partial \rho_k^3}{\partial y} & 0 & 0 & \frac{\partial \rho_k^3}{\partial z} & 0 & 0 & 1 \\ \frac{\partial \rho_k^4}{\partial x} & 0 & 0 & \frac{\partial \rho_k^4}{\partial y} & 0 & 0 & \frac{\partial \rho_k^4}{\partial z} & 0 & 0 & 1 \end{bmatrix} \quad (7)$$